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ASYMPTOTIC SOLUTIONS FOR THE EXPANSION OF GAS INTO VACUUM

By E. H. Wedemeyer

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ABERDEEN PROVING GROUND, MARYLAND

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RDT & E Project No. 1A222901A201

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ASYMPTOTIC SOLUTIONS FOR THE EXPANSION OF GAS INTO VACUUM

ABSTRACT

When a spherical or cylindrical gas cloud expands into vacuum, the resultant flow becomes asymptotically self-similar and independent of the original cloud diameter. An analytic solution for the asymptotic density profiles, given by Mirels and Mullen [4] is re-examined and a new solution proposed, which gives better agreement with numerical computations.

Some applications of the asymptotic solutions are discussed.

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1. INTRODUCTION

The investigations reported here were initiated by a paper "Translational Equilibration of Wires Exploded in Vacua," by O. H. Zinke et al [1]*. The authors of this paper made use of an unsteady spherical expansion of ionized gas in order to infer the temperature of the gas from the characteristics of the expansion. The method described by Zinke can be a powerful tool, provided that the unsteady flow resulting from an expanding gas cloud is understood theoretically. It is believed that the theoretical solution presented here straightens out some discrepancies observed by Zinke. Further remarks on this are given in section 5. The solution for the unsteady cylindrical expansion can also be applied to a related problem, the expansion of a hypersonic jet into vacuum.

In the above mentioned and other cases, the main interest is in the asymptotic flow field after long times. For large times the density distribution of the expanding gas cloud approaches an asymptotic similarity form, which is independent of the original diameter of the cloud.

It can be shown, however, that the shape of the asymptotic density profile depends on the initial density distribution of the gas before expansion. In the following an attempt is made to derive approximate solutions for the asymptotic density profiles for the case that the initial density distribution is uniform. Expansions with plane, cylindrical and spherical symmetry will be considered.

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* *Numbers in brackets denote references listed on page 25.*

2. ASYMPTOTIC NATURE OF THE FLOW

In the following it is assumed that the flow is isentropic and that the gas can be treated as a continuum; i.e., the mean free path is very small compared with the initial diameter of the gas cloud. Then the equations of motion for flows of cylindrical ($\sigma = 1$) or spherical ($\sigma = 2$) symmetry are respectively:

$$\frac{\partial \bar{\rho}}{\partial \bar{t}} + \frac{\partial \bar{\rho}\,\bar{u}}{\partial \bar{r}} + \sigma \frac{\bar{\rho}\bar{u}}{\bar{r}} = 0 \tag{1a}$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{u}}{\partial \bar{r}} + \frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial \bar{r}} = 0 \tag{1b}$$

$$(\bar{p}/p_0) = (\bar{\rho}/\rho_0)^{\gamma} \tag{1c}$$

ρ_0 and p_0 are taken as initial density and pressure if the density is initially uniform, or as the initial values at $r = 0$ if the density is not uniform. It is assumed that the gas is at rest initially and inclosed within a cylinder or sphere of radius R_0.

For the following we introduce nondimensional quantities:

$$\rho = \frac{\bar{\rho}}{\rho_0} \quad u = \frac{\bar{u}}{a_0} \quad r = \frac{\bar{r}}{R_0} \quad t = \frac{\bar{t}a_0}{R_0} \tag{2}$$

where a_0 is the sound speed of the quiescent gas and found by

$$a_0^2 = \gamma \frac{p_0}{\rho_0} \tag{3}$$

With these nondimensional quantities the equations of motion are:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial r} + \sigma \frac{\rho u}{r} = 0 \tag{4a}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \rho^{\gamma-2}\frac{\partial \rho}{\partial r} = 0 \tag{4b}$$

For the case that the density is initially uniform, the boundary conditions are:

$$\begin{aligned} &\text{at } t = 0;\ 0 < r < 1: && \rho = 1,\ u = 0 \\ &\qquad\quad 1 < r: && \rho = 0 \\ &\text{at } t \geq 0,\ r = 0: && u = 0 \end{aligned} \tag{5}$$

For the case that $\sigma = 0$, the Equations (4) together with boundary conditions (5) describe the one-dimensional (plane) expansion of a slab of gas extending either from $r = -1$ to $r = +1$ with $r = 0$ a plane of symmetry or from $r = 0$ to $r = 1$ with a fixed wall at $r = 0$. In a few cases, exact solutions of Equations (4a, 4b) can be obtained. These solutions which we shall discuss in the next paragraph, will serve as a guide to construct approximate asymptotic solutions for the more general case.

Here we are concerned with the asymptotic nature of the flow after long times; i.e., for times large compared to R_0/a_0. For times which are large in this sense, the original gas cloud has expanded many times and the pressure term $\rho^{\gamma-2} \frac{\partial \rho}{\partial r}$ in Equation (4b) becomes relatively small. After neglecting the pressure term Equation (4b) is solved by

$$u = r/t \tag{6}$$

With the solution (6) for u, Equation (4a) has the solution:

$$\rho t^{\sigma+1} = \text{const } f(r/t) \tag{7}$$

where $f(r/t)$ is an arbitrary function of the one variable r/t. It can be seen from Equation (6) and (7) that the flow becomes asymptotically self-similar; i.e., the density and velocity distributions at different times are similar and differ only by scaling factors. While this similarity form of Equations (6) and (7) means a certain simplification of the mathematical treatment, the problem of finding the particular solution for $f(r/t)$ is still very difficult for the following reason. Since any solution of the form (6) and (7) solves the equations of motion asymptotically, any of these solutions represents a possible asymptotic flow, each one belonging to different initial conditions. Thus, one has to relate a given initial density distribution to a corresponding $f(r/t)$ and it seems that this can be done exactly only by computing the whole flow history from the beginning of the expansion up to large times where the flow becomes self-similar. A computation of this sort can be performed numerically, using the finite step method of characteristics. It must be considered, however, that any finite step computation may become increasingly inaccurate when carried out for too many steps, due to the accumulation of errors. The number of steps necessary in order to reach the asymptotic similarity-solution must be enormously large. In spite of this, numerical computations for uniform initial densities and various values of γ and σ have been carried out by several authors [2,3] and it appears that the results of these computations are surprisingly accurate.

3. ANALYTICAL SOLUTION

A first attempt toward an analytical solution has been made by Mirels and Mullen [4], however the solution does not show satisfactory agreement with numerical results [2,3].

The solution proposed here is obtained by only slight modification of the view of Mirels and Mullen [4] but it gives, nevertheless, far better agreement with numerical results. The procedure used to derive an analytical expression for the asymptotic flow consists in finding a valid generalization of certain exact solutions. As has been mentioned before, in a few cases the Equations (4a), (4b) can be solved exactly. These cases are:

1. The one-dimensional ($\sigma = 0$) expansion of an initially uniform slab of gas.

2. The similarity flows for $\sigma = 0, 1, 2$. These flows are strictly self-similar at all times, but the initial density-distribution is not uniform.

The characteristics solution for one-dimensional expansion from uniform initial conditions (case 1) is given in [5] (Courant and Friedrichs) and is discussed in greater detail in [6,7]. In the asymptotic limit, this solution simplifies to:

$$\rho t = \frac{\gamma-1}{\sqrt{\pi}} \frac{\Gamma(N+\frac{1}{2})}{\Gamma(N)} \left[1 - \left(\frac{\gamma-1}{2}\frac{r}{t}\right)^2\right]^{\frac{3-\gamma}{2(\gamma-1)}} \tag{8a}$$

$$u = r/t \tag{8b}$$

where $N = \frac{\gamma+1}{2(\gamma-1)}$ and Γ denotes the gamma-function.

The similarity flows have been investigated in References [8] and [9]. If $R(t)$ is the position of the leading edge of the expansion, one can define a similarity variable

$$y = r/R(t) \tag{9}$$

so that the gas is confined in $0 \le y \le 1$.

Assuming for the self-similar flow a form $\rho = R^{-(\sigma+1)} f(y)$; $v = \frac{dR}{dt} \cdot g(y)$ and substituting into Equations (4a, b) one obtains

$$\rho = R^{-(\sigma+1)} (1 - y^2)^{\frac{1}{\gamma-1}} \tag{10}$$

and

$$u = \frac{dR}{dt} \cdot y \tag{11}$$

where R satisfies the differential equation

$$\frac{dR}{dt} = \frac{1}{\sqrt{\sigma+1}} \cdot \frac{2}{\gamma-1} [1 - R^{-(\sigma+1)(\gamma-1)}]^{\frac{1}{2}} \tag{12}$$

The constants of integration are so determined, that for

$$t = 0: \quad R = 1; \frac{dR}{dt} = 0 \ .$$

In the asymptotic limit one finds

$$R = \frac{1}{\sqrt{\sigma+1}} \cdot \frac{2}{\gamma-1} \cdot t \tag{13}$$

and Equations (10) and (11) become asymptotically:

$$\rho t^{\sigma+1} = \left(\sqrt{\sigma+1}\ \frac{(\gamma-1)}{2}\right)^{\sigma+1}\left[1 - \left(\sqrt{\sigma+1} \ . \ \frac{\gamma-1}{2}\ \frac{r}{t}\right)^2\right]^{\frac{1}{\gamma-1}} \tag{14a}$$

$$u = r/t \tag{14b}$$

A comparison of Equation (8) and Equation (14) shows that both formulas are quite similar and of the form:

$$\rho t^{\sigma+1} = D\left[1 - \left(\frac{r}{Vt}\right)^2\right]^B \tag{15a}$$

$$u = r/t \tag{15b}$$

where D, V and B are constants. Thus, we assume, in accordance with Mirels and Mullen [4], that the asymptotic solution for uniform initial density and $\sigma \neq 0$ is also of the form (15); so that it remains only to determine the constants D, V and B. If one of the three constants is given, say V, then the other two can be determined from the conditions, that mass and total energy are conserved. In our nondimensional variables these two conditions can be expressed as:

$$\int_0^{Vt} \rho r^\sigma \, dr = \frac{1}{\sigma+1} \quad ; \tag{16}$$

$$\int_0^{Vt} \frac{\rho}{2} u^2 r^\sigma \, dr = \frac{1}{\sigma+1} \cdot \frac{1}{\gamma(\gamma-1)} \quad . \tag{17}$$

Equation (16) equates the mass of the expanded gas to the mass at $t = 0$ and Equation (17) does the same for the energy. Note, that for $t \to \infty$ the thermal energy vanishes so that only the integral over the kinetic energy density $\rho/2 \; u^2$ is taken at the left side of Equation (17), while for $t = 0$ the kinetic energy is zero and the thermal energy per unit mass is:

$$C_v T_o = \frac{a_o^2}{\gamma(\gamma-1)} \quad .$$

Introducing the new variable $y = \frac{r}{Vt}$ and inserting (15a, b) into (16) and (17) gives:

$$DV^{\sigma+1} \int_0^1 (1 - y^2)^B y^\sigma \, dy = \frac{1}{\sigma+1} \quad ; \tag{18}$$

$$DV^{\sigma+3} \int_0^1 (1 - y^2)^B y^{\sigma+2} dy = \frac{2}{\sigma+1} \cdot \frac{1}{\gamma(\gamma-1)} \quad . \tag{19}$$

The integrals in Equations (18) and (19) can be transformed into Eulerian integrals of the first kind and expressed in terms of gamma functions. It is, e.g.,

$$\int_0^1 (1 - y^2)^B y^\sigma \, dy = \frac{1}{2} \frac{\Gamma(B+1)\,\Gamma\left(\frac{\sigma+1}{2}\right)}{\Gamma\left(B+1 + \frac{\sigma+1}{2}\right)} \quad . \tag{20}$$

Solving Equation (18) and (19) for B and D one obtains:

$$B = \left[\frac{\gamma(\gamma-1)}{2} V^2 - 1\right]\left(\frac{\sigma+1}{2}\right) - 1 \quad ; \tag{21}$$

$$D = \frac{1}{V^{\sigma+1}} \cdot \frac{2}{\sigma+1} \cdot \frac{\Gamma(B+1 + \frac{\sigma+1}{2})}{\Gamma(B+1)\Gamma(\frac{\sigma+1}{2})} \quad . \tag{22}$$

Now only one constant, V, remains to be determined and at this point the present analysis differs from Mirels and Mullen [4]. From Equation (15a) it can be seen, that V is the velocity of the leading edge of the expansion (in units of a_o).

Now Mirels and Mullen maintain that for uniform initial density the velocity of the leading edge should be the same as for one-dimensional flow, and therefore $V = \frac{2}{\gamma-1}$, regardless of the value of σ. In other words, the geometry, whether spherical or cylindrical, does not influence the leading edge velocity. The arguments given in favor of this are difficult and not fully convincing, but even if they were true they would not demand for V a value of $\frac{2}{\gamma-1}$. It is understood that Equation (15a) can be only an approximate formula which must not reflect every detail of an exact solution. It could be, that a minute fraction of the total mass "escapes" with the leading edge velocity $\frac{2}{\gamma-1}$ and that the bulk of the remaining mass expands with an "effective" leading edge velocity less than that. In order to have the best approximation, valid for the bulk of the mass, it would then be advisable to pick a smaller value for V. This is also suggested by Equation (14a) for the similarity flow.

Here one observes that the geometry does have an influence on the leading edge velocity expressed by the geometrical factor $\frac{1}{\sqrt{\sigma+1}}$. If the geometrical factor $\frac{1}{\sqrt{\sigma+1}}$ has an influence in the case of similarity flow, it is reasonable to assume that it has an analogous influence in the case of uniform initial density. This would suggest:

$$V = \frac{1}{\sqrt{\sigma+1}} \cdot \frac{2}{\gamma-1} \quad , \tag{23}$$

for both similarity and uniform initial density flow.

A more rigorous derivation of (23) is given in Appendix A.

With (23) inserted into (21) and (22) the constants B and D are:

$$B = \left(\frac{1}{\gamma-1}\right) - \left(\frac{\sigma+1}{2}\right) \quad , \tag{24}$$

$$D = \left(\frac{2}{\sigma+1}\right)\left(\sqrt{\sigma+1}\,\frac{(\gamma-1)}{2}\right)^{\sigma+1} \frac{\Gamma\left(B+1+\frac{\sigma+1}{2}\right)}{\Gamma(B+1)\,\Gamma\left(\frac{\sigma+1}{2}\right)} \quad , \tag{25}$$

and from Equation (15a),

$$\rho t^{\sigma+1} = D\left[1 - \left(\frac{\sqrt{\sigma+1}}{2}\,(\gamma-1)\,\frac{r}{t}\right)^2\right]^B \quad . \tag{26}$$

For $\sigma = 0$ Equation (8a) is recovered from Equation (26); i.e., Equation (26) is exact for $\sigma = 0$. For $\sigma \neq 0$ the formula (26) is different from Mirels and Mullen's, who obtain larger values for the exponent B and the factor D as a result of the larger V values they assumed. In fact, for $\sigma \geq 1$ the exponents B obtained by Mirels and Mullen are even larger than the exponent for similarity flow, which is $\frac{1}{\gamma-1}$ (Equation 14a). A consequence of this is that the resultant density distribution is less full than the corresponding distribution for similarity flow. This is an odd result, considering that the uniform density profile is fuller initially and one ought to expect that it also remains fuller than the similarity profile after expansion. The exponents obtained by Mirels and Mullen are so large (e.g. for $\sigma = 2$, $\gamma = 7/5$; B = 8 instead B = 1 following from Equation 24) that they essentially cut off the density profile at a distance inside of the leading edge which is tantamount to our supposition that only a minute fraction of the mass actually achieves a velocity close to that of the leading edge. In other words, the large values of B tend to partly compensate for the prohibitively large leading edge velocity assumed by Mirels and Mullen.

The values here observed for B (Equation 24) are always smaller and therefore the density profiles fuller, than for the corresponding similarity flow.

Apparently the formula (26) for the density profile becomes nonsensical for $B < 0$, or

$$\frac{1}{\gamma-1} < \frac{\sigma+1}{2} \tag{27}$$

Now, it is interesting to note that inequality (27) describes just those cases which are physically meaningless. To see this, we recall that $\gamma = \frac{N+2}{N}$ where N is the number of internal degrees of freedom of the gas. Therefore, $\frac{1}{\gamma-1} = \frac{N}{2}$. On the other hand $(\sigma+1)$ is the number of dimensions of the space into which the gas expands. If this is $(\sigma+1) = n$, then inequality (27) states: $N < n$, which obviously cannot happen, since even a hypothetical gas must have at least as many internal as external degrees of freedom.

The density factor D of Equation (25) has a simple physical meaning: it gives, apart from the time factor $t^{\sigma+1}$, the density at the origin $r = 0$. The factor D is plotted in Figure 1 and Figure 2 for $\sigma = 1, 2$ versus γ. For comparison, the corresponding curves obtained by Mirels and Mullen [4]

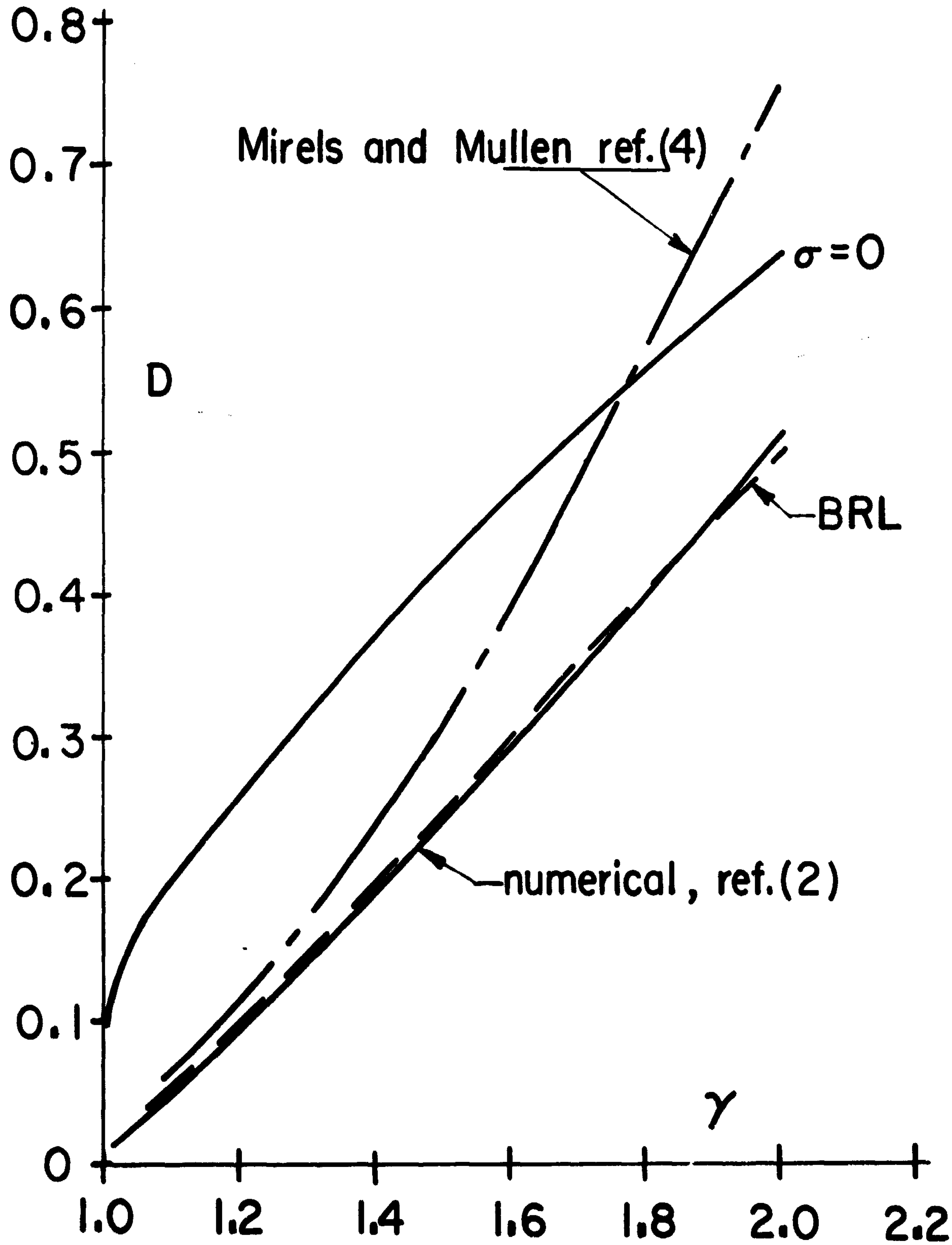

FIGURE 1

Density factor for $\sigma = 1$.

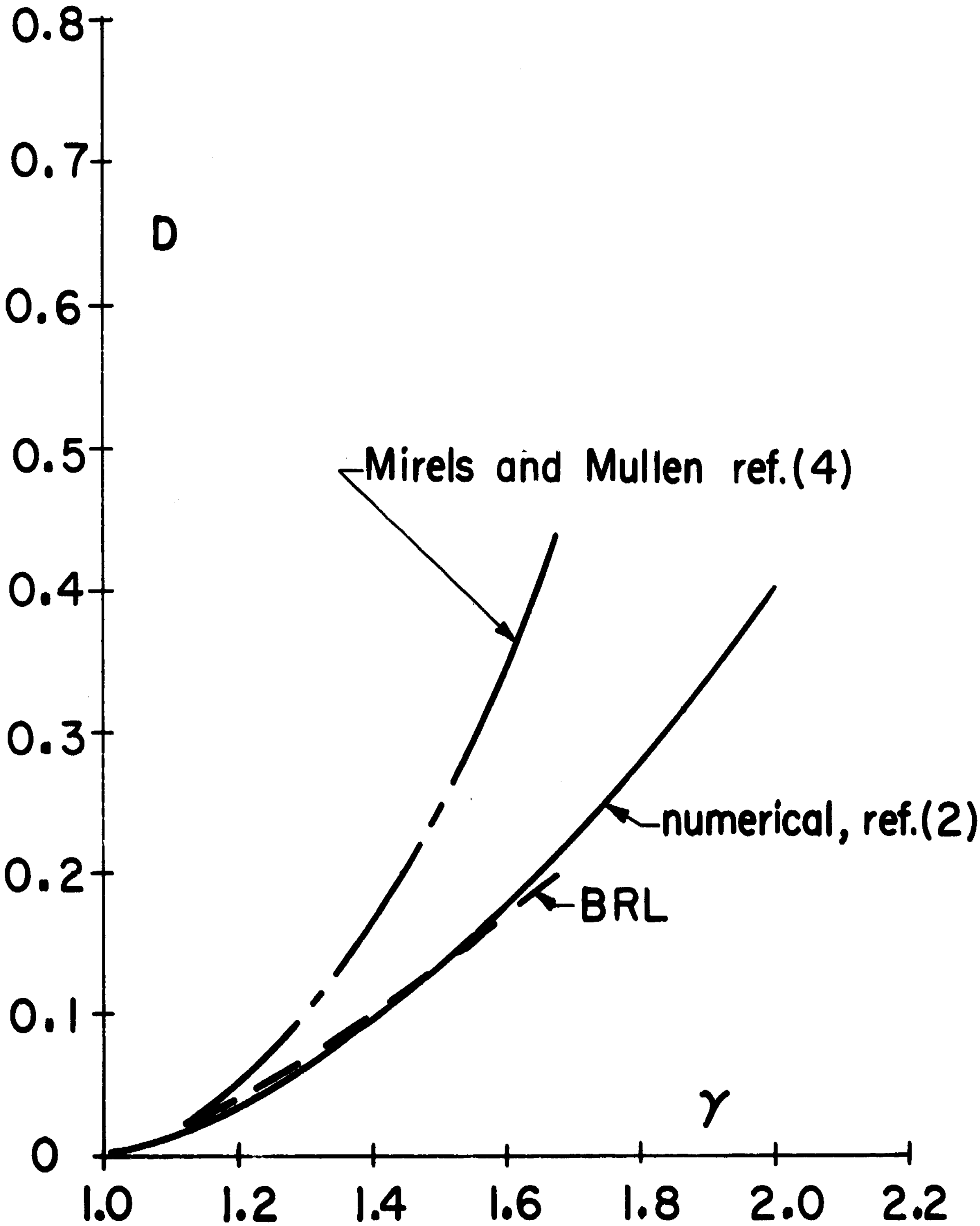

FIGURE 2

Density factor for $\sigma = 2$.

and the results of numerical computations by Greifinger [2] are plotted in the same diagram. Also the curve for $\sigma = 0$ is shown in Figure 1. For this case the results of Mirels and Mullen [4], Greifinger [2] and the present analysis agree with each other and the exact solution. For $\sigma = 1, 2$ the results of the present analysis are in very good agreement with the numerical data [2].

Another check against numerical results is given in Figure 3, which shows the density profile for $\sigma = 1$, $\gamma = 5/3$ in comparison with the results of Mirels and Mullen [4] and numerical data computed at NRL [3]. All profiles are normalized by dividing through the density at $r = 0$. Again, the agreement between the present analysis and the numerical computation is very good. In Figure 2 the values for D according to Equation (24) are plotted only for $\gamma \leq 5/3$. For larger γ-values the exponent B (Equation 24) becomes negative and the formula (26) is no longer applicable. One might suspect that the approximate solution (Equation 26) becomes less accurate on approaching $\gamma = 5/3$.

4. LATERAL EXPANSION OF A HYPERSONIC JET BOUNDED BY VACUUM

If the lateral flow velocity u of the expanding jet is small compared with its axial velocity V_0, then the equations of motion for steady flow reduce to Equations (1) with $\bar{t}$ replaced by $\frac{\bar{x}}{V_0}$, where $\bar{x}$ is the axial distance from the nozzle. Using the nondimensional quantities defined in Equation (2) and defining a nondimensional axial distance $x = \frac{\bar{x}}{R_0}$, one

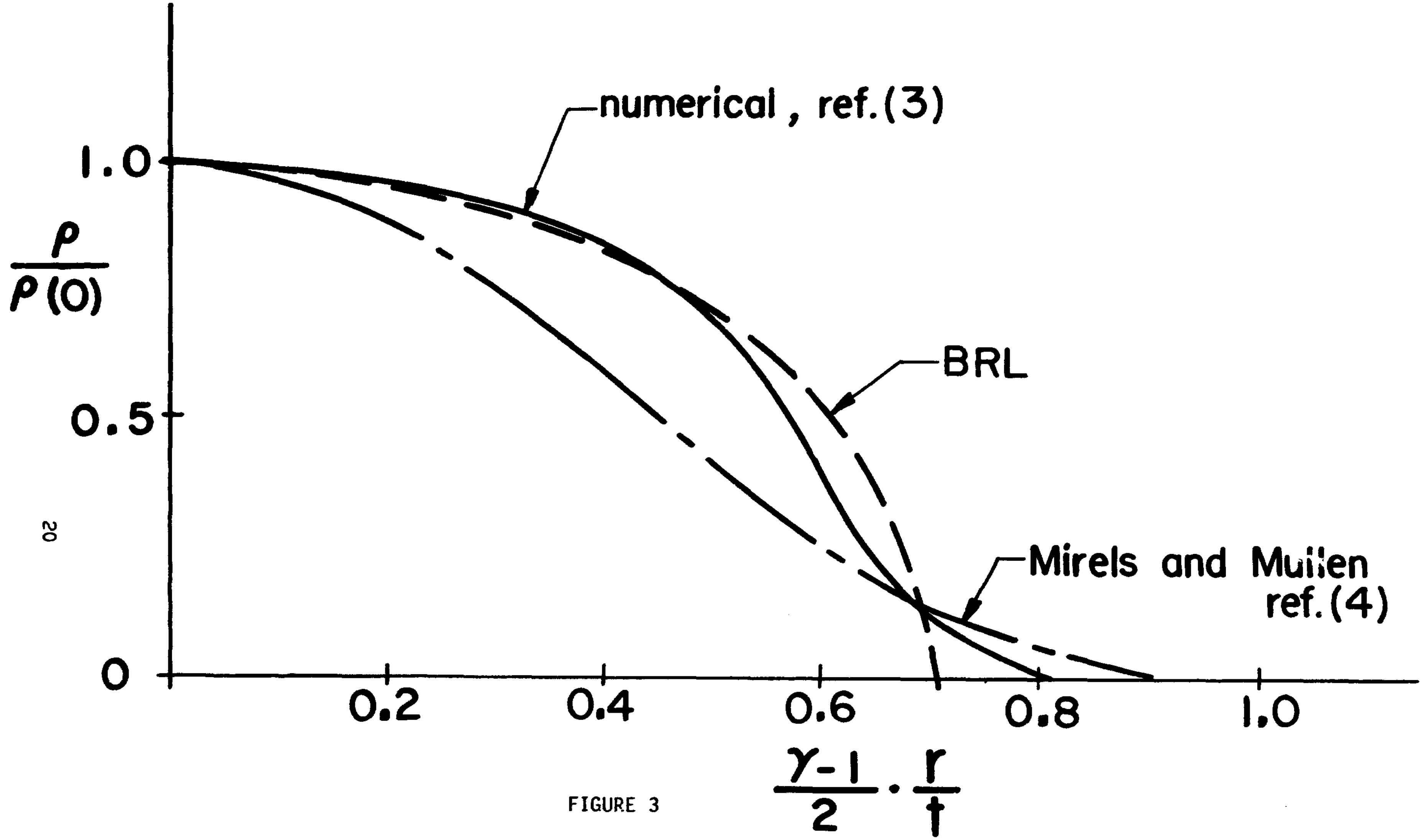

FIGURE 3

Asymptotic density profile for $\sigma = 1$, $\gamma = \frac{5}{3}$

has to replace t in Equations (4) by $\frac{xa_o}{V_o} = \frac{x}{M_o}$. Thus, the solution for the density distribution of the jet is obtained from Equation (26) for $\sigma = 1$ and substituting $\frac{x}{M_o}$ for t. This solution is valid for $M_o \gg 1$.

5. MEASUREMENT OF PLASMA TEMPERATURE

In their paper, "Translational Equilibration of Wires Exploded in Vacua," [1] the authors describe a method for measuring the temperature of ionized gas with the aid of a drift tube. The method, which is described in detail in [1], is essentially as follows: The plasma source, in this case an exploding wire, is located at one end of a long evacuated drift tube, while an electrode is placed at its opposite end. It is assumed that the wire is heated uniformly by a short current pulse. After the short heating period the gaseous, ionized wire material starts to expand without being heated subsequently. The ion flux, after reaching the electrode, gives rise to a time dependent current in the electrode circuit which can be observed. From the observed current distribution the temperature of the original plasma cloud can be inferred.

In evaluating the experimental data Zinke et al. [1] assumed, the plasma would expand like a collision-free gas; i.e., according to free molecular flow. On the basis of this assumption, it was impossible to fit the theoretical prediction to the experimental data unless a hypothetical "containment-time" was introduced. In other words, the shape of the observed current-time distribution was quite different from the shape of any of the possible theoretical distributions. The theoretical current distributions for free molecular flow are given by

$$j(\bar{t}) = \frac{K}{\bar{t}^4} \exp\left(-\frac{m}{2kT}\frac{\bar{r}^2}{\bar{t}^2}\right) \tag{28}$$

where $\bar{t}$ is the time elapsed from the explosion of the wire, $\bar{r}$ the distance between the explosion and the electrode, T the temperature of the original plasma cloud and K a constant related to the total number of ions.

The distributions Equation (28) form a two-parameter class of curves with parameters K and T. None of these curves fit the experimental data. A satisfactory fit could be achieved only after introducing a third parameter, H, the "containment-time." Assuming the plasma did not start to expand at time t = 0, i.e., immediately after heating, but at a later time, t = H, the distribution Equation (28) would be altered to:

$$j(\bar{t}) = \frac{K}{(\bar{t}-H)^4} \exp\left[-\frac{m}{2kT}\frac{\bar{r}^2}{(\bar{t}-H)^2}\right] \quad . \tag{29}$$

With properly chosen K, T, H, Zinke could obtain a reasonable fit to all experimental data. However, the physical meaning of the containment-time, H, remained completely unclear. Obviously, the assumption of free molecular flow is not correct for the high initial densities encountered in the wire explosion. In continuum expansion the energy of the gas molecules is redistributed as a result of collisions. This may significantly change the flow and the current distribution observed at the electrode of the drift tube. The theoretical current distribution for continuum flow is proportional to $\rho u = \rho \cdot \frac{\bar{r}}{\bar{t}}$ with ρ given by Equation (26) and $\sigma = 2$. Thus:

$$j = \frac{K.\bar{r}}{\bar{t}^4}\left[1 - \left(\frac{\sqrt{3}(\gamma-1)}{2}\frac{\bar{r}}{a_n \bar{t}}\right)^2\right]^B \tag{30}$$

Here $\bar{r}$ is the distance between the electrode and the plasma source (exploding wire) and $\bar{t}$ the time elapsed from the explosion.

When K and a_0 were chosen properly, a good fit could be obtained to the experimental data of Zinke without the assumption of a containment-time. From a_0 the temperature can be determined by: $a_0^2 = \gamma \frac{kT}{m}$.

The values for the temperature obtained from this fit were about 10^5 °K. These values are more likely correct than those obtained by Zinke of about 10^6 °K.

APPENDIX A

Introducing a new dependent variable ρ^* by

$$\rho = (\rho^*)^{\sigma+1} \tag{31}$$

and substituting (31) into Equations (4a, b) gives the following set of equations for ρ^*:

$$\frac{\partial \rho^*}{\partial t} + \frac{\partial \rho^* u}{\partial r} + \frac{\sigma}{\sigma+1} \rho^* \left[\frac{u}{r} - \frac{\partial u}{\partial r}\right] = 0 \tag{32a}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + (\sigma+1)\rho^{*(\gamma-2)(\sigma+1)+\sigma} \frac{\partial \rho^*}{\partial r} = 0 \tag{32b}$$

Let us now consider the factor $(\frac{u}{r} - \frac{\partial u}{\partial r})$ in the last term of Equation (32a). This factor is exactly zero for the similarity solution, as can be seen from Equation (11).

For the case of uniform initial density this factor becomes asymptotically zero, since $u \to \frac{r}{t}$. It is also zero near the center $r = 0$, which follows from the boundary conditions (5) according to which $u = c_1 r + c_2 r^2 + \ldots$ We therefore assume, that the average effect of the last term of Equation (32a) is small and to a certain approximation it can be neglected altogether.

With the further transformations:

$$u^* = \frac{u}{\sqrt{\sigma+1}} \quad ; \quad t^* = t\sqrt{\sigma+1} \qquad (33)$$

$$(\gamma^* - 2) = (\gamma-2)(\sigma+1) + \sigma$$

we obtain by substituting into (32a,b):

$$\frac{\partial\rho^*}{\partial t^*} + \frac{\partial\rho^* u^*}{\partial r} = 0 \qquad (34a)$$

$$\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial r} + (\rho^*)^{(\gamma^*-2)} \frac{\partial\rho^*}{\partial r} = 0 \qquad (34b)$$

From (34a,b) we observe, that ρ^*, u^* satisfy the equations of motion (4a,b) for $\sigma = 0$ with γ^* replacing γ. ρ^*, u^* also satisfy the same boundary conditions as ρ, u for the case of uniform initial density. The asymptotic solution, therefore, can be obtained from (8a,b) with ρ, u, t, γ replaced by ρ^*, u^*, t^*, γ^*; i.e.,

$$\rho^* t^* \sim \left[1 - \left(\frac{\gamma^*-1}{2}\frac{r}{t^*}\right)^2\right]^{\frac{3-\gamma^*}{2(\gamma^*-1)}} , \qquad (35a)$$

$$u^* = r/t^* \quad . \qquad (35b)$$

Taking the $(\sigma+1)$ power of Equation (35a) and substitution of (31) and (33) into (35a,b) gives:

$$\rho t^{\sigma+1} = \text{const}\left[1 - \left(\sqrt{\sigma+1}\;\frac{(\gamma-1)}{2}\frac{r}{t}\right)^2\right]^{\left(\frac{1}{\gamma-1} - \frac{\sigma+1}{2}\right)} , \qquad (36a)$$

$$u = r/t \quad . \qquad (36b)$$

The constant in Equation (36a) can now be re-adjusted so as to satisfy the mass conservation. The solution (36a) then is identical with Equation (26).

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13. ABSTRACT

When a spherical or cylindrical gas cloud expands into vacuum, the resultant flow becomes asymptotically self-similar and independent of the original cloud diameter. An analytic solution for the asymptotic density profiles, given by Mirels and Mullen[4] is re-examined and a new solution proposed, which gives better agreement with numerical computations.

Some applications of the asymptotic solutions are discussed.

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